



**NAMIBIA UNIVERSITY  
OF SCIENCE AND TECHNOLOGY  
FACULTY OF HEALTH AND APPLIED SCIENCES**

**DEPARTMENT OF MATHEMATICS AND STATISTICS**

<b>QUALIFICATION:</b> Bachelor of Science in Applied Mathematics and Statistics	
<b>QUALIFICATION CODE:</b> 08BSHM	<b>LEVEL:</b> 8
<b>COURSE CODE:</b> AOR802S	<b>COURSE NAME:</b> APPLIED OPERATIONS RESEARCH
<b>SESSION:</b> NOVEMBER 2022	<b>PAPER:</b> THEORY
<b>DURATION:</b> 3 HOURS	<b>MARKS:</b> 240 (To be Converted to 100%)

<b>FIRST OPPORTUNITY EXAMINATION QUESTION PAPER</b>	
<b>EXAMINER</b>	PROF. S. A. REJU
<b>MODERATOR:</b>	PROF. O. D. MAKINDE

<b>INSTRUCTIONS</b>
<ol style="list-style-type: none"><li>1. Attempt ALL the questions.</li><li>2. All written work must be done in blue or black ink and sketches must be done in pencil.</li><li>3. Use of COMMA is not allowed as a DECIMAL POINT.</li></ol>

**PERMISSIBLE MATERIALS**

1. Non-programmable calculator without a cover.

**THIS QUESTION PAPER CONSISTS OF 4 PAGES** (including this front page)

**QUESTION 1 [50 MARKS]**

- (a) Discuss Game Reduction by Dominance procedure. (4 Marks)  
 (b) Simplify by using reduction by dominance the game defined by the following payoff matrix, showing progressively the reduced pay-off matrix: (18 Marks)

		B		
		a	b	c
A	A	2	-1	-5
	B	4	-4	2
	C	3	-3	-8
	D	2	-5	-4

- (c) Distinguish between game players' pure and mixed strategies, with clear characteristics of the latter. (6 Marks)  
 (d) Ezra has a 270-gallon capacity home heating oil tank, presently empty, meant to store oil against the next winter. Consider the following winter heating oil quantity needed and the oil prices during probable four levels of winter severity: (22 Marks)

WINTER SEVERITY	OIL STORAGE NEEDED	OIL PRICES PER GALLON
Mild Winter (MW)	125 Gallons	N\$1.00
Average Winter (AW)	180 Gallons	N\$1.85
Severe Winter (SW)	240 Gallons	N\$2.00
Prolonged Winter (PW)	270 Gallons	N\$3.00

Formulate a game model and employ the Minimax criterion technique to determine the gallons of oil Ezra should stockpile at the current price of N\$1 per gallon to avoid oil wastage and to maximise his saving.

**QUESTION 2 [53 MARKS]**

- (a) Two suspects A and B have been apprehended for a crime and are in cells in Tsumeb police station, with no means of communicating with each other. The prosecutor has separately told them the following:

*If you confess and agree to testify against the other suspect, who does not confess, the charges against you will be dropped and you will go scot-free. If you do not confess but the other suspect does, you will be convicted and the prosecution will seek the maximum sentence of three years. If both of you confess, you will both be sentenced to two years in prison. If neither of you confesses, you will both be charged with misdemeanours and will be sentenced to one year in prison.*

Selecting Suspect A as the row player in a 2-person game, construct the game payoff matrix and each suspect's payoff matrix. (12 Marks)

- (b) (i) Determine what the two suspects should do and discuss fully why. (6 Marks)
- (ii) Discuss the implication of the dominant strategy for each player (or prisoner). (5 Marks)
- (c) Consider a construction firm that is deciding to specialise in building High School blocks or Elementary School blocks or a combination of both. The construction company must submit a bid proposal, which costs money to prepare, and there are no guarantees that it will be awarded the contract. If the company bids on the high school, it has a 35% chance of getting the contract, and it expects to make \$162,000 net profit. However, if the company does not get the contract, it loses \$11,500. If the company bids on the elementary school, there is a 25% chance of getting the contract, and it would net \$140,000 in profit. However, if the company does not get the contract, it will lose \$5,750.
- (i) What should the construction company do? (14 Marks)
- (ii) How sensitive to the estimate of the probability of the award of a contract is the decision (i):
- in either to build the High School or the Elementary School blocks? (6.5 Marks)
  - to the net profit for each case, if awarded the contract? (9.5 Marks)

### QUESTION 3 [53 MARKS]

- (a) Provide a comprehensive definition of a Decision tree and hence diagrammatically show its basic characteristic components. (14 Marks)
- (b) Using the problem in Question 2(c) above, provide the Fold-Back method tree for its solution. (14 Marks)
- (c)
- (i) What is the Kendall's classification of Queuing Systems? (5 Marks)
- Discuss specifically the M/M/1 queuing system and the process  $N(t)$  describing its state at time  $t$  as a birth-death process. Provide its state independent parameter equations and define its Traffic Intensity. (3 Marks)
- (ii) Consider a drive-in banking service modelled as an M/M/1 queuing system with customer arrival rate of 2 per minute. It is desired to have fewer than 5 customers line up 99% of the time. How fast should the service rate be? (6 Marks)
- (iii) Trucks arrive at garage for a stop-over service according to a Poisson process at a rate of one per every 13 minutes, and the garage service time is an exponential rate variable with mean 9 minutes.
- (iiia) Find the average number  $L$  of trucks, the average time  $W$  a truck spends in the garage, and the average time  $W_q$  a truck spends in waiting for service. (5 Marks)
- (iiib) Due to increased traffic, suppose that the arrival rate of the trucks increases by 5%. Find the corresponding changes in  $L$ ,  $W$ , and  $W_q$ . (5 Marks)
- (iiic) Discuss your observations. (1 Mark)

**QUESTION 4 [84 MARKS]**

- (a) Consider a winning bid of \$5.4 million to construct a new plant for a major manufacturer and the manufacturer needs the plant to go into operation within 40 weeks. Below is the list of the various project activities. The third column provides important additional information for coordinating the scheduling of the project crews.

Activity	Activity Description	Immediate Predecessors	Estimated Duration
A	Excavate	—	2 weeks
B	Lay the foundation	A	4 weeks
C	Put up the rough wall	B	10 weeks
D	Put up the roof	C	6 weeks
E	Install the exterior plumbing	C	4 weeks
F	Install the interior plumbing	E	5 weeks
G	Put up the exterior siding	D	7 weeks
H	Do the exterior painting	E, G	9 weeks
I	Do the electrical work	C	7 weeks
J	Put up the wallboard	F, I	8 weeks
K	Install the flooring	J	4 weeks
L	Do the interior painting	J	5 weeks
M	Install the exterior fixtures	H	2 weeks
N	Install the interior fixtures	K, L	6 weeks

- (i) Define Critical Path Method (CPM) and Project Evaluation and Review Technique (PERT). (3.5 Marks)
- (ii) Sketch the project network diagram for the above project. (16 Marks)
- (iii) Distinguish between crashing a project and a project activity. Hence obtain the crash costs per week saved for each activity from the following investigative time-cost trade-off data. (37 Marks)

Activity	Time		Cost		Activity	Time		Cost	
	Normal	Crash	Normal	Crash		Normal	Crash	Normal	Crash
A	2 weeks	1 week	\$180,000	\$ 280,000	I	7 weeks	5 weeks	\$210,000	\$ 270,000
B	4 weeks	2 weeks	\$320,000	\$ 420,000	J	8 weeks	6 weeks	\$430,000	\$ 490,000
C	10 weeks	7 weeks	\$620,000	\$ 860,000	K	4 weeks	3 weeks	\$160,000	\$ 200,000
D	6 weeks	4 weeks	\$260,000	\$ 340,000	L	5 weeks	3 weeks	\$250,000	\$ 350,000
E	4 weeks	3 weeks	\$410,000	\$ 570,000	M	2 weeks	1 week	\$100,000	\$ 200,000
F	5 weeks	3 weeks	\$180,000	\$ 260,000	N	6 weeks	3 weeks	\$330,000	\$ 510,000
G	7 weeks	4 weeks	\$900,000	\$1,020,000					

- (iv) Discuss your observations. (4 Marks)
- (b) Consider a flow network with a directed graph with three vertices and three arcs, described as follows: The first arc from vertex(1) to vertex(2) has capacity 3 and the cost 1; the second arc from vertex(1) to vertex(3) has capacity 5 and the cost 4, and the third arc from vertex(2) to vertex(3) has capacity 4 and the cost 2.
- (i) Sketch the flow network. (12 Marks)
- (ii) State the matrices for the arc capacities, the arc costs and the demand function for the vertices. (11.5 Marks)